Exercise 274

For the following exercises, use properties of logarithms to write the expressions as a sum, difference, and/or product of logarithms.

$$\log_4 \frac{\sqrt[3]{xy}}{64}$$

Solution

There are three properties of logarithms to know.

$$\log(ab) = \log a + \log b \tag{1}$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b \tag{2}$$

$$\log a^b = b \log a \tag{3}$$

Use properties (2) and (3).

$$\log_4 \frac{\sqrt[3]{xy}}{64} = \log_4 \sqrt[3]{xy} - \log_4 64$$

$$= \log_4 (xy)^{1/3} - \log_4 64$$

$$= \frac{1}{3} \log_4 xy - \log_4 64$$

$$= \frac{1}{3}(\log_4 x + \log_4 y) - \log_4 64$$

$$= \frac{1}{3}(\log_4 x + \log_4 y) - 3$$

Note that $\log_4 64 = 3$ because $4^3 = 64$.